Quantisation of free associative dynamical systems. Quantisation ideals

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In my talk I'll discuss a new approach to the problem of quantisation of dynamical systems, introduce the concept of quantisation ideals and provide meaningful examples. Traditional quantisation theories start with classical Hamiltonian systems with variables taking values in commutative algebras and then study their non-commutative deformations, such that the commutators of observables tend to the corresponding Poisson brackets as the (Planck) constant of deformation goes to zero. I am proposing to depart from dynamical systems (or maps) defined on a free associative algebra $\mathfrak{A}$. In this approach the quantisation problem is reduced to description of two-sided ideals $\mathfrak{J} \subset \mathfrak{A}$ satisfying two conditions: the ideals have to be invariant with respect to the dynamics of the system and they define commutation relations in the quotient algebras $\mathfrak{A}_{\mathfrak{J}} = \mathfrak{A} \diagup \mathfrak{J}$. Such ideals are called the quantisation ideals. Surprisingly, this idea works rather efficiently and in a number of cases I have been able to quantise the system, i.e. to find commutation relations consistent with the dynamics of the system. A Poisson structure is not assumed in this apprach, but if it is known, then it helps to find quantisation ideals corresponding the deformation of the Poisson strcture.

To illustrate this approach I'll consider the quantisation problem for ODEs on free associative algebra, including N–th Novikov equations and corresponding finite KdV hierarchy. If time permits, I'll discuss quantisation of the Bogoyavlensky family of integrable N–chains (in the case $N = 1$ it is a well known Volterra chain):

$$\frac{du_n}{dt} = \sum_{k=1}^{N}(u_{n+k}u_n - u_nu_{n-k}), \qquad n \in \mathbb{Z}, \tag{1}$$

quantisation of their symmetries and modifications. In particular, I will show that odd degree symmetries of the Volterra chain admit two quantisations, one of them corresponds to known quantisation of the Volterra chain, and another one is new and it is not deformational.

The talk is based on:

AVM, *Quantisation ideals of nonabelian integrable systems*, arXiv:2009.01838, 2020 (Published in Russ. Math. Surv. v.75:5, pp 199-200, 2020)

V.M.Buchstaber and AVM, *KdV hierarchies and quantum Novikov's equations*, arXiv:2109.06357v1, 2021.